Inhomogeneous nucleation in quark hadron phase transition

P. Shukla, A.K. Mohanty, and S.K. Gupta
Nuclear Physics Division, Bhabha Atomic Research Centre,
Trombay, Mumbai 400 085, India

Marcelo Gleiser
Department of Physics and Astronomy, Dartmouth College,
Hanover, NH 03755, USA

Abstract

The effect of subcritical hadron bubbles on a first-order quark-hadron phase transition is studied. These subcritical hadron bubbles created due to thermal fluctuations introduce a finite amount of phase mixing (quark phase mixed with hadron phase) even at and above the critical temperature. For sufficiently strong transitions, as is expected to be the case for the quark-hadron transition, we show that the amount of phase mixing at the critical temperature remains much below the percolation threshold. Thus, as the system cools below the critical temperature, the transition proceeds through the nucleation of critical-size hadron bubbles from a metastable quark-gluon phase (QGP) within an inhomogeneous background populated by an equilibrium distribution of subcritical hadron bubbles. The inhomogeneity of the medium is incorporated consistently by modeling the subcritical bubbles as Gaussian fluctuations, resulting in a large reduction of the nucleation barrier for the critical bubbles. Using the corrected nucleation barrier, we estimate the amount of supercooling for different parameters controlling the phase transition.
PACS number(s): 12.38.Mh, 64.60.Qb, 05.70.Fh, 25.75-q, 98.80.Cq

I. INTRODUCTION

The dynamics of a first order phase transition from quark gluon plasma (QGP) to hadron matter has been studied in detail in the framework of homogeneous nucleation theory [1–6]. In this picture, the transition is initiated by the nucleation of critical size hadron bubbles from a supercooled metastable QGP phase. These hadron bubbles can grow against surface tension converting the QGP phase into the hadron phase as the temperature drops below the critical temperature, T_C. This is indeed the case for a sufficiently strong first order transition, where the assumption of a homogeneous background of QGP is justified at the time when the nucleation begins. However, for a weak enough transition, the QGP phase may not remain in a pure homogeneous state even at $T = T_C$ due to pre-transitional phenomena. We describe it through an effective thermodynamic potential as a function of an order parameter ϕ. For temperatures much above T_C, the matter is in the pure QGP phase with the effective potential exhibiting one minimum at ϕ=0. As the plasma expands and cools to some temperature T_1, an inflection point is developed away from the origin which on further cooling separates into a maximum at ϕ=ϕ_m and a local minimum at $\phi = \phi_h$ corresponding to hadron phase. At $T = T_C$, the potential is degenerate with a barrier that separates the two phases. This is the general behaviour of the effective potential describing a first-order phase transition. "Pre-transitional phenomena" refers to nonperturbative dynamical effects above T_C in the range $T_C \leq T \leq T_1$. Such phenomena are known to occur in several areas of condensed matter physics, as in the case of isotropic to nematic phase transition in liquid crystals [7], and are also expected in the cosmological electroweak phase transition [8]. Thermal fluctuations about the equilibrium state occur with probability $\exp[-F(T)/T]$, where $F(T)$ is the free energy of the particular fluctuation. Large amplitude fluctuations will have a finite probability of populating other accessible state around the new minimum at $\phi = \phi_h$. Although these fluctuations which are in the form of subcritical hadron bubbles will always shrink and finally disappear, there will always be some non-zero number density of hadron bubbles at a given temperature which will attain a full equilibrium value, if the cooling rate of the medium is sufficiently slow. Thus, the quark gluon phase is mixed with subcritical hadron bubbles and the medium can be called inhomogeneous. If the fraction of these subcritical bubbles is large, the two phases are completely mixed and the phase transition will take place through percolation [9]. For a stronger quark-hadron transition, the density of subcritical hadron bubbles remains below the percolation threshold. Therefore, the plasma has to supercool and the phase transition will be initiated through the nucleation of critical size hadron bubbles. However, instead of a (near) homogeneous background, the metastable QGP phase will have considerable phase mixing, which will influence the dynamics of the transition. In Refs. [10–12], an approximate method was developed to compute the equilibrium fraction of subcritical bubbles. In Ref. [13], this method was applied to compute corrections to the homogeneous nucleation rate due to the presence of subcritical fluctuations in the background. Following this latter work, we will assume that the system is close enough to the regime described by homogeneous nucleation so that we can still distinguish the two phases. As will be shown subsequently, this assumption is reasonable in the case of the quark-hadron phase transition. The aim of the present work is to estimate the amount of phase mixing and its effect on supercooling during a first order quark-hadron phase transitions ranging from very weak to very strong.

The paper is organized as follows. In the next section, we begin with the discussion of a quartic double-well potential used to describe the dynamics of a first-order quark-hadron phase transition. The parameters of the potential are obtained in terms of relevant physical quantities such as critical temperature, surface tension and correlation length. In section III, we estimate the equilibrium fraction of subcritical hadron bubbles from very weak to strong first order phase transitions. We also estimate the reduction in the nucleation barrier by incorporating the presence of subcritical bubbles in the medium. Using this reduced barrier, we study nucleation and supercooling in section IV. Finally, we present our conclusions in section V.

II. PARAMETERIZATION OF THE EFFECTIVE POTENTIAL

We consider a general form of the potential (or equivalently, the homogeneous part of the Helmholtz free energy density) to study the quark-hadron phase transition in terms of a real scalar order parameter ϕ given by

$$V(\phi, T) = a(T)\,\phi^2 - b\,T\,\phi^3 + c\,\phi^4, \tag{1}$$

where b and c are positive constants. The potential has two minima, one at $\phi_q = 0$ and the other at $\phi_h = (3bT + \sqrt{9b^2T^2 - 32ac})/8c$, which in our case will represent quark and hadron phases respectively. These phases are separated by a maximum defined by $\phi_m = (3bT - \sqrt{9b^2T^2 - 32ac})/8c$. At $T = T_C$,

$$V(\phi_q, T_C) = V(\phi_h, T_C) = 0, \tag{2}$$

having the required degeneracy. The second part of Eq. (2) yields,

$$a(T_C) = b^2T_C^2/4c, \quad \phi_h(T_C) = bT_C/2c \;\text{ and }\; \phi_m(T_C) = bT_C/4c. \tag{3}$$

Using these relations, the barrier height at T_C can be obtained as

$$V_b = b^4T_C^4/256c^3. \tag{4}$$

Therefore, if the parameter c is kept fixed, b can be varied to characterize a wide spectrum of very weak to very strong first-order phase transitions. The transition is strong enough for large V_b and very weak or close to second order as $V_b \to 0$. In the following, we relate the parameters b and c to the surface tension and the correlation length in the quark phase. The surface tension can be defined as the one dimensional action given by,

$$\sigma = \int dx \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + V(\phi)\right]. \tag{5}$$

Under the limit $\Delta = |V(0) - V(\phi_h)| \to 0$, the potential is nearly degenerate and the surface tension can be expressed as [14]

$$\begin{aligned}\sigma &= \int_0^{\phi_h} d\phi\sqrt{2V(\phi)},\\ &= \frac{\sqrt{2}}{48}\frac{b^3T_C^3}{c^{5/2}}.\end{aligned} \tag{6}$$

Similarly, the correlation length around the quark phase is obtained using $\xi_q = 1/\sqrt{V''(\phi)}|_{\phi=0}$ $= 1/\sqrt{2a(T)}$. At the critical temperature, using Eq. (3), we get

$$\xi_q(T_C) = \frac{\sqrt{2c}}{bT_C}. \tag{7}$$

From Eqs. (6) and (7) we get

$$c = \frac{1}{12\xi_q^3\sigma}, \qquad b^2 = \frac{1}{6\xi_q^5\sigma T_C^2}, \tag{8}$$

in terms of the values of σ and ξ_q at T_C. The barrier height V_b can now be written as

$$V_b = \frac{3}{16}\frac{\sigma}{\xi_q(T_C)}. \tag{9}$$

Thus, the barrier height is proportional to the ratio σ/ξ_q. The transition becomes very weak as σ decreases and ξ_q increases. Here, we fix $\xi_q = 0.5$ fm at $T = T_C$ and vary σ to investigate phase transitions with different strengths. The temperature dependence of a is deduced by equating the depth of the second minimum with the the pressure difference ΔP between the two phases at all temperatures. This yields an equation

$$\begin{aligned} \Delta P &= p_h - p_q \\ &= V(0) - V(\phi_h) \\ &= -\left(a(T) - bT\phi_h + c\phi_h^2\right)\phi_h^2 \end{aligned} \tag{10}$$

which is solved to get the parameter $a(T)$ which also gives the temperature dependence of ξ_q. The surface tension will also have small temperature dependence which we ignore as we are not going too far from the critical temperature. Thus, we have parameterized the free-energy density in terms of the surface tension, correlation length, critical temperature and equation of state, which can be obtained from lattice QCD calculations. The bag equation of state which is a good depiction of the lattice results is used to calculate the quark/hadron pressure $p_{q/h}$ as follows

$$p_q = a_q T^4, \qquad p_h = a_h T^4 - B, \tag{11}$$

where $B = (a_q - a_h)\, T_C^4$ is the bag constant. The quark phase is assumed to consist of a massless gas of u and d quarks and gluons, while the hadron phase contains massless pions. Thus, the coefficients a_q and a_h are given by $a_q = 37\pi^2/90$ and $a_h = 3\pi^2/90$. The critical temperature is taken as $T_C = 160$ MeV.

Fig. 1 shows the plot of $V(\phi)$ as a function of ϕ at three different temperatures for a typical value of $\sigma = 30$ MeV/fm^2 and $\xi_q(T_C) = 0.5$ fm. At $T = T_C$, the potential is degenerate with a large barrier that separates the two phases. Below T_C, the phase $\phi = \phi_h$ has lower free-energy density, and the QGP phase becomes metastable. Above T_C, the potential has a metastable mimima at $\phi = \phi_h$ (hadron phase) as long as T remains below T_1. The temperature T_1 [at which $\phi_h = \phi_m$ and $9b^2T_1^2 = 32a(T_1)\, c$] can be obtained analytically by solving Eq. (10) as,

$$T_1 = \left[\frac{B}{B - \frac{27}{16}V_b}\right]^{1/4} T_C. \tag{12}$$

It may be mentioned here that the dynamics of the phase transition has also been studied in Ref. [15] using a different form of the potential which has been parameterized as a fourth order polynomial in the energy density [1]. This form is unsuitable over a wide range of temperatures due to the persistence of metastability at much above and below T_C.

III. MODEL FOR LARGE-AMPLITUDE FLUCTUATIONS

We closely follow the work of Refs. [11–13] to estimate the equilibrium density distribution of subcritical hadron bubbles by modeling them as Gaussian fluctuations with amplitude ϕ_A and radius R

$$\phi_{q\to h}(r) = \phi_A e^{-r^2/R^2} \quad \text{and} \quad \phi_{h\to q}(r) = \phi_A \left(1 - e^{-r^2/R^2}\right). \tag{13}$$

The amplitude ϕ_A is the value of the field at the bubble's core away from the quark phase. For smooth interpolation between the two phases in the system, $\phi_A \geq \phi_m$. The free energy of a given configuration can then be found by using the general formula [14],

$$F = \int d^3r \left[\frac{1}{2}(\nabla\phi(r))^2 + V(\phi(r))\right]. \tag{14}$$

Using Eq. (13) and Eq. (1) in Eq. (14) we get

$$F_{q\to h} = \alpha_h\, R + \beta_h\, R^3 \quad \text{and} \quad F_{h\to q} = \alpha_q\, R + \beta_q\, R^3, \tag{15}$$

where α_h, β_h, α_q and β_q are given by

$$\alpha_h = \alpha_q = \frac{3\sqrt{2}}{8}\pi^{3/2}\phi_A^2, \qquad \beta_h = \left[\frac{\sqrt{2}a}{4} - \frac{\sqrt{3}bT}{9}\phi_A + \frac{c}{8}\phi_A^2\right]\pi^{3/2}\phi_A^2 \tag{16}$$

and

$$\begin{aligned}\beta_q = {} & \left(\frac{\sqrt{2}}{4} - 2\right) a\pi^{3/2}\phi_A^2 - \left(-\frac{\sqrt{3}}{9} - 3 + \frac{3\sqrt{2}}{4}\right) bT\pi^{3/2}\phi_A^3 \\ & + \left(\frac{1}{8} + \frac{3\sqrt{2}}{2} - \frac{4\sqrt{3}}{9} - 4\right) c\pi^{3/2}\phi_A^4.\end{aligned} \tag{17}$$

It may be mentioned here that $\alpha_h (= \alpha_q)$ is positive and is much greater than $\beta_{h(q)}$. Therefore, the free energy grows linearly for smaller values of R. Further, hadron bubbles of all configurations will be subcritical as long as $\beta_{h(q)}$ is positive. At $T = T_C$, both β_h and β_q are positive for all amplitudes. However, below T_C, β_h may become negative for some values of ϕ_A. For such configurations, the free energy has a maximum at $R_m = \sqrt{\alpha_h/3\beta_h}$ and these bubbles are not strictly subcritical. The same is true for β_q above T_C. We thus restrict the amplitudes ϕ_A to the range where $\beta_{h(q)}$ is positive. If not exactly the same, the limits of integration ϕ_{min} and ϕ_{max} for ϕ_A are found to be quite close to ϕ_m and ϕ_h respectively.

A. Equilibrium fraction of subcritical bubbles

There will be fluctuations from quark to hadron phase and hadron phase to quark phase. To obtain the number density n_A of subcritical bubbles we define the distribution function $f \equiv \partial^2 n_A/\partial R\partial\phi_A$ where $f(R, \phi_A, t)dRd\phi_A$ is the number density of bubbles with radius between R and $R + dR$ and amplitude between ϕ_A and $\phi_A + d\phi_A$ at time t. It satisfies the Boltzmann equation [12,13]

$$\frac{\partial f(R,\phi_A,t)}{\partial t} = -|v|\frac{\partial f}{\partial R} + (1-\gamma)G_{q\to h} - \gamma G_{h\to q}. \tag{18}$$

The first term on the RHS is the shrinking term. Here, $|v|$ is the shrinking velocity, which we assume to be given by the velocity of sound ($= 1/\sqrt{3}$) in a massless gas. The second term is the nucleation term where G is the nucleation distribution function defined as $\Gamma = \int dR d\phi G$. Here $\Gamma_{q\to h}$ is the nucleation rate per unit volume of subcritical bubbles from the quark phase to the hadron phase. Similarly $\Gamma_{h\to q}$ is the corresponding rate from the hadron phase to the quark phase. The factor γ is defined as the fraction of volume in the hadron phase and is obtained by summing over subcritical bubbles of all amplitudes and radii within this phase. The Gibbs nucleation rate distribution function per unit volume is taken as [11,14]

$$G = A\,T^4\,e^{-F(R,\phi_A)/T}, \tag{19}$$

where A is of the order of ~ 1 [14].

If the equilibration time scale is smaller than the expansion time scale of the system, we can obtain the equilibrium number density of subcritical bubbles by solving Eq. (18) with $\partial f/\partial t = 0$. For a quark-hadron phase transition in the early universe [16,17], the cooling rate is slow due to the high initial temperature, whereas in the case of QGP produced during relativistic heavy ion collisions the cooling is faster. In this case, it is possible that the density distribution of the subcritical bubbles will not attain full equilibrium. However, in this work we assume an equilibrium situation so that the present results on the fraction of subcritical bubbles and phase mixing can be considered as the upper limit of this analysis. Using the boundary condition $f(R\to\infty) = 0$, we get the equilibrium distribution given by

$$f(R,\phi_A,T) = (1-\gamma)\,W_S(R,\phi_A,T) - \gamma\,W_T(R,\phi_A,T), \tag{20}$$

where

$$W_S(R,\phi_A,T) = (A/|v|)T^4\int_R^\infty e^{-(\alpha_h R' + \beta_h R'^3)/T}dR',$$
$$W_T(R,\phi_A,T) = (A/|v|)T^4\int_R^\infty e^{-(\alpha_q R' + \beta_q R'^3)/T}dR'. \tag{21}$$

The equilibrium fraction γ of volume occupied by subcritical bubbles is given by,

$$\gamma(\phi_{min},\phi_{max},R_{\rm min},R_{\rm max}) = \int_{\phi_{min}}^{\phi_{max}}\int_{R_{min}}^{R_{\rm max}}\frac{4\pi}{3}R^3 f(R,\phi_A,T)dRd\phi_A, \tag{22}$$

which is solved to give

$$\gamma = \frac{I_S}{1+I_S+I_T}, \tag{23}$$

where

$$I_{S(T)} = \int_{\phi_{min}}^{\phi_{max}}\int_{R_{\rm min}}^{R_{\rm max}}\frac{4\pi}{3}R^3 W_{S(T)}(R,\phi_A,T)dRd\phi_A\ . \tag{24}$$

Here, ϕ_{min} and ϕ_{max} define the range within which both β_h and β_q are positive. R_{min} is the smallest radius of the subcritical bubbles taken as ξ_q, the correlation length of the fluctuations. The R integration should be carried out over all bubbles with radii from $R_{min} = \xi_q$

to $R_{max} = \infty$. For very weak transitions, both α and β are very small and the R integration may not have good convergence. However, we found that the γ value is maximized when R_{max} is about 3 to 4 fm. Therefore, we use R_{max}=3.5 fm. This is a reasonable choice as bubbles with $R \sim \xi_q$ will be statistically dominant and larger fluctuations have larger free energy and are exponentially suppressed.

Fig. 2 shows the plot of subcritical hadron fraction γ as a function of σ at $T = T_C$ and at a fixed value of $\xi_q(T_C) = 0.5$ fm. The γ has also been estimated (dashed curve) with the assumption that for a degenerate potential $G_h \simeq G_q$, as has been used in Ref. [13]. This assumption is valid only for the configuration for which $\phi_A = \phi_h$. However, when we include other configurations in the range ϕ_{min} to ϕ_{max}, the integral I_T turns out to be always higher than I_S at T_C. Therefore, γ obtained using $G_h \neq G_q$ is always lower than that when the approximation $G_h = G_q$ is used. In both cases, the value of γ increases with decreasing σ in other words as the transition becomes weak. For a large range of surface tension, the fraction γ is small. As per lattice QCD calculations [18], σ may lie in the range of 5 MeV/fm^2 to 15 MeV/fm^2. There could be 10% to 20% phase mixing corresponding to these σ values. This fraction is still below the percolation threshold ($\gamma \leq 3$). If $\gamma \geq 0.3$, then the two phases will mix completely, the mean-field approximation for the potential breaks down, and the phase transition will proceed through percolation [9,12]. However, in the present case, the phase transition has to proceed through the formation of critical size hadron bubbles from a supercooled metastable QGP phase. Since the QGP phase is no longer homogeneous, the dynamics of the phase transition will be quite different from what is expected on the basis of homogeneous nucleation theory [13] and we refer to it as the inhomogeneous nucleation.

We would also like to mention here that the present results are in complete disagreement with the findings of Ref. [15], where a large fraction of sub-critical hadron phase is obtained at and above T_C. This scenario is highly unrealistic and probably could be due to the choice of the potential parameterization which shows a metastable hadron phase much above T_C. Therefore, they found a finite fraction of hadron phase at temperatures as high as twice T_C. Further, the value of γ strongly depends on how the shrinking term is incorporated in the calculation. In our case, it is proportional to the gradient $(\partial f/\partial R)$ that appears in the kinetic equation (18) in a natural way, whereas in [15], a specific assumption is made to take into account the shrinking of the hadronic volume.

B. The total free energy of subcritical bubbles and the nucleation barrier

The nucleation rate in the standard theory [5,6] which neglects phase mixing, is given by

$$I \simeq AT^4 \, e^{-F_{\rm C}/T}. \tag{25}$$

Here $F_{\rm C}$ is free energy needed to form a critical bubble in the homogeneous metastable background. For an arbitrary thin-walled spherical bubble of radius R and amplitude $\phi_{\rm thin} \lesssim \phi_h$, the free energy of the bubble takes the well-known form

$$F_{\rm thin}(R) = -\frac{4\pi}{3} R^3 \Delta V + 4\pi R^2 \sigma. \tag{26}$$

ΔV is defined as the difference in free-energy density between the background medium and the bubble's interior. For a homogeneous background (metastable) we can write,

$$\Delta V \equiv \Delta V_0 = V(0) - V(\phi_h). \tag{27}$$

If there is significant phase mixing in the background metastable state, its free energy is no longer $V(0)$. One must also account for the free energy density of the nonperturbative large amplitude fluctuations. Following Ref. [13], we write the free energy density of the metastable state as $V(0) + \mathcal{F}_{\rm sc}$, where $\mathcal{F}_{\rm sc}$ is the extra free energy density which can be estimated from the density distribution of subcritical bubbles as follows:

$$\begin{aligned}\mathcal{F}_{\rm sc} &\approx \int_{\phi_{min}}^{\phi_{max}} \int_{R_{\rm min}}^{R_{\rm max}} F_h(R, \phi_A, T)\ f(R, \phi_A, T) dR d\phi_A, \\ &= (1-\gamma) \int_{\phi_{min}}^{\phi_{max}} \int_{R_{\rm min}}^{R_{\rm max}} F_h\, W_S\, dR d\phi_A - \gamma \int_{\phi_{min}}^{\phi_{max}} \int_{R_{\rm min}}^{R_{\rm max}} F_h\, W_T\, dR d\phi_A. \end{aligned} \tag{28}$$

Once we know the hadronic fraction γ and the free energy F_h for a bubble of a given radius R and amplitude ϕ_A, we can estimate the free-energy density correction due to the presence of Gaussian subcritical bubbles.

Since, for a critical size bubble, *i.e.* $\partial F/\partial R|_{R_C} = 0$, we can now use Eq. (26) to obtain the free energy needed to form a thin-wall critical bubble in a background of subcritical bubbles,

$$F_C = \frac{4\pi}{3} \sigma R_C^2, \quad R_C = \frac{2\sigma}{\Delta V_0 + \mathcal{F}_{\rm sc}}. \tag{29}$$

For a very strong first-order phase transition, the subcritical bubbles are suppressed ($\mathcal{F}_{\rm sc} \to 0$), and both F_C and R_C approach the homogeneous background expression. However, in the presence of subcritical bubbles, extra free energy becomes available in the medium which reduces the nucleation barrier. In other words, the extra background energy enhances the nucleation of the critical bubbles. To illustrate this, we have plotted F_C/T and γ as a function of T/T_C in Figs. 3 to 5 with σ values of 50 MeV/fm^2, 30 MeV/fm^2 and 10 MeV/fm^2, respectively, which are widely used in the literature. As evident, with decreasing temperature, the nucleation barrier decreases and subcritical hadron fraction γ increases. The reduction in barrier height due to $\mathcal{F}_{\rm sc}$ or we can say due to γ is more significant for lower values of σ corresponding to a weaker transition. Thus, unlike the homogeneous case, the nucleation will begin earlier if the corrected nucleation barrier is used, which would reduce the amount of supercooling. The time evolution of the temperature and the supercooling are discussed in the next section.

IV. NUCLEATION AND SUPERCOOLING

As mentioned before, the background metastable state is inhomogeneous due to subcritical hadron bubbles. It is now possible to study the kinetics of the nucleation of the critical hadron bubbles using the corrected nucleation rate, obtained in the previous section. In the present work, the prefactor in the nucleation rate is taken as AT^4 [see Eq. (25)]. In our previous work, [4], we have used a prefactor derived by Csernai and Kapusta [2] for a dissipative QGP. In Ref. [19], Ruggeri and Friedman derived a prefactor for a non-dissipative QGP. Recently, using a more general formalism we have also derived a prefactor [20] which has both dissipative and non-dissipative components corresponding to Ref. [2]

and Ref. [19], respectively. However, for consistency with the subcritical formalism, we use a more generic form $I_0 = AT^4$ with A is a constant of order unity which is used in many studies of quark-hadron phase transition [see for example, Refs. [16,17]].

Using the nucleation rate $I(T)$, the fraction h of space which has been converted to hadron phase due to nucleation of critical bubbles and their growth can be calculated. If the system cools to T_C at a proper time τ_c, then at some later time τ the fraction h [2] is

$$h(\tau) = \int_{\tau_c}^{\tau} d\tau' I\left(T(\tau')\right)\left[1 - h(\tau') - \gamma\left(T(\tau')\right)\right] V(\tau', \tau). \tag{30}$$

Here, $V(\tau', \tau)$ is the volume of a critical bubble at time τ which had been nucleated at an earlier time τ'; this takes into account the bubble growth. The factor $[1 - h(\tau) - \gamma\left(T(\tau)\right)]$ accounts for the available space for new bubbles to nucleate. It should be noticed here that h is the hadronic fraction due to the critical hadron bubbles and their growth. The γ is fraction due to subcritical bubbles as obtained in previous section and the time dependence of which solely comes through the temperature. The model for bubble growth is simply taken as [21]

$$V(\tau', \tau) = \frac{4\pi}{3}\left(R_C(T(\tau')) + \int_{\tau'}^{\tau} d\tau'' v(T(\tau''))\right)^3, \tag{31}$$

where $v(T) = 3[1 - T/T_c]^{3/2}$ is the velocity of the bubble growth at temperature T [22]. The evolution of the energy density in 1+1 dimensions is given by

$$\frac{de}{d\tau} + \frac{\omega}{\tau} = 0. \tag{32}$$

The energy density e, enthalpy density ω and the pressure p in pure QGP and hadron phases are given by the bag model equation of state. In the transition region, the e and ω at a time τ can be written in terms of hadronic fraction as

$$\begin{aligned} e(\tau) &= e_q(T) + \left[e_h(T) - e_q(T)\right]\left[h(\tau) + \gamma(\tau)\right], \\ \omega(\tau) &= \omega_q(T) + \left[\omega_h(T) - \omega_q(T)\right]\left[h(\tau) + \gamma(\tau)\right]. \end{aligned} \tag{33}$$

Equations (30), (32), and (33) are solved to get the temperature as a function of time in the mixed phase [4]. The initial temperature is taken as $T_0 = 250$ MeV and initial time as $\tau_0 = 1$ fm/c. This gives the time at which the critical temperature $T_C = 160$ MeV is reached as $\tau_C = 3.8$ fm/c.

Figure 6 shows the temperature variation as a function of proper time at $\sigma = 50$ MeV/fm^2. As the system cools below T_C, the nucleation barrier decreases and γ increases. If only homogeneous nucleation (dashed curve) is considered, the system will supercool up to 0.945 T_C. At this temperature, the hadronic fraction γ has reached 10 % (See Fig. 3), which corrects the amount of supercooling (solid curve) by about ∼10 % (to 0.95 T_C). Figure 7 shows a similar study at $\sigma = 30$ MeV/fm^2. Since the nucleation barrier reduces with reducing σ, the system supercools only up to 0.98 T_C. The hadronic fraction γ corresponding to this value is $\sim 12 - 13$ % (See Fig. 4) which reduces the amount of supercooling by about $\sim$ 20% (to 0.984 T_C). If we take an even smaller σ around 10 MeV/fm^2 the supercooling will be very small (upto $\sim 0.997 T_C$ to $0.998 T_C$), the corresponding γ will be about 15-20%

of the total volume (See Fig. 5), which will further reduce the amount of supercooling. Since the precise calculation of such a small amount of supercooling is difficult with our nucleation calculation we have not plotted this curve. Thus, it is shown that, for smaller σ the fraction γ builds faster as the system cools. But we never encounter γ greater than 3 as the supercooling is smaller which will be further reduced by the subcritical bubble correction.

V. CONCLUSIONS

We have investigated the effect of phase mixing due to the subcritical hadron bubbles on a first order quark-hadron phase transition. We estimated the equilibrium fraction of these subcritical bubbles around the critical temperature for a wide range of very weak to very strong first-order phase transitions. With a reasonable set of values for the surface tension and correlation length (as obtained from lattice QCD calculations), we found that the phase mixing is small at $T = T_C$, building up as the temperature drops further. We have shown that, since the supercooling is not large, the system does not mix beyond the percolation threshold and we can still describe the dynamics of phase transition on the basis of homogeneous nucleation with the added correction. Thus, we computed the nucleation rate of a supercooled quark-gluon plasma with inhomogeneities due to subcritical hadronic fluctuations. We found an enhancement of the nucleation rate and a reduction of the amount of supercooling. In this study, we have assumed that the equilibration time-scale for subcritical fluctuations is much larger than the cooling time-scale of the system. This may be the case for a quark-hadron phase transition in the early universe where the expansion rate is quite slow. In the case of QGP produced at RHIC and LHC, the cooling rate may be much faster than cosmological time-scales and the subcritical bubbles density distribution may not attain full equilibration. We are presently investigating this effect in more detail. However, by assuming thermal equilibrium, we can use our results to provide an upper bound on the fraction of subcritical hadron bubbles their effect on the supercooling.

ACKNOWLEDGMENTS

MG is partially supported by a National Science Foundation Grant PHYS-9453431.

REFERENCES

[1] L. P. Csernai and J. I. Kapusta, Phys. Rev. D **46**, 1379 (1992).

[2] L. P. Csernai and J. I. Kapusta, Phys. Rev. Lett. **69**, 737 (1992).

[3] E. E. Zabrodin, L. V. Bravina, L. P. Csernai, H. Stocker and W. Griener, Phys. Lett, B423, 373 (1998).

[4] P. Shukla, S. K. Gupta, and A. K. Mohanty, Phys. Rev. C**59**, 914 (1999) and hep-ph/9904345.

[5] J. S. Langer, Ann. Phys. (NY) **41**, 108 (1967); *ibid.* **54**, 258 (1969).

[6] J. S. Langer and L. A. Turski, Phys. Rev. A **8**, 3230 (1973)

[7] T. Stinton III and J. Lister, Phys. Rev. Lett. **25**, 503 (1970).; H. Zink and W. H. de Jeu, Mol. Cryst. Liq. Cryst. **124**, 287 (1985).

[8] M. Gleiser and E. W. Kolb, Phys. Rev. Lett. **69**, 1304 (1992); M. Gleiser and M. Trodden, hep-ph/9911380.

[9] D. Stauffer and A. Aharony, *Introduction to Percolation Theory*, Taylor & Francis, Washington, DC, 1992.

[10] M. Gleiser, E. W. Kolb and R. Watkins, Nucl. Phys. **B** 364, 411 (1991).

[11] G. Gelmini and M. Gleiser, Nucl. Phys. B **419**, 129 (1994).

[12] M. Gleiser, A. F. Heckler, and E. W. Kolb, Phys. Lett. (1995).

[13] M. Gleiser and A. F. Heckler, Phys. Rev. Lett **76**, 180 (1996).

[14] A. Linde, Nucl. Phys. **B216**, 421 (1983).

[15] B. K. Agarwal and S. Digal, Phys. Rev. **C60**, 074007 (1999).

[16] G. M. Fuller, G. J. Mathews and C. R. Alcock, Phys. Rev. D **37**, 1380 (1988).

[17] B. S. Meyer, C. R. Alcock, and G. J. Mathews Phys. Rev. D **43**, 1079 (1991).

[18] Y. Iwasaki, K. Kanaya, L. Karkkainen, K. Rummukainen, and T. Yoshie, Phys. Rev. D**49**, 3540 (1994).

[19] Franco Ruggeri and William A. Friedman, Phys. Rev. D **53**, 6543 (1996).

[20] P. Shukla, A. K. Mohanty, S. K. Gupta (hep-ph/0005219).

[21] S. Weinberg, Astrophys. J. **168,** 175 (1971).

[22] J. C. Miller and O. Pantano, Phys. Rev. D **40**, 1789 (1989); **42**, 3334 (1990).

Figure Captions

Fig. 1 Plot of the effective potential as a function of order parameter at, below and above T_c.

Fig. 2 Plot of subcritical hadronic fraction γ as a function of surface tension σ.

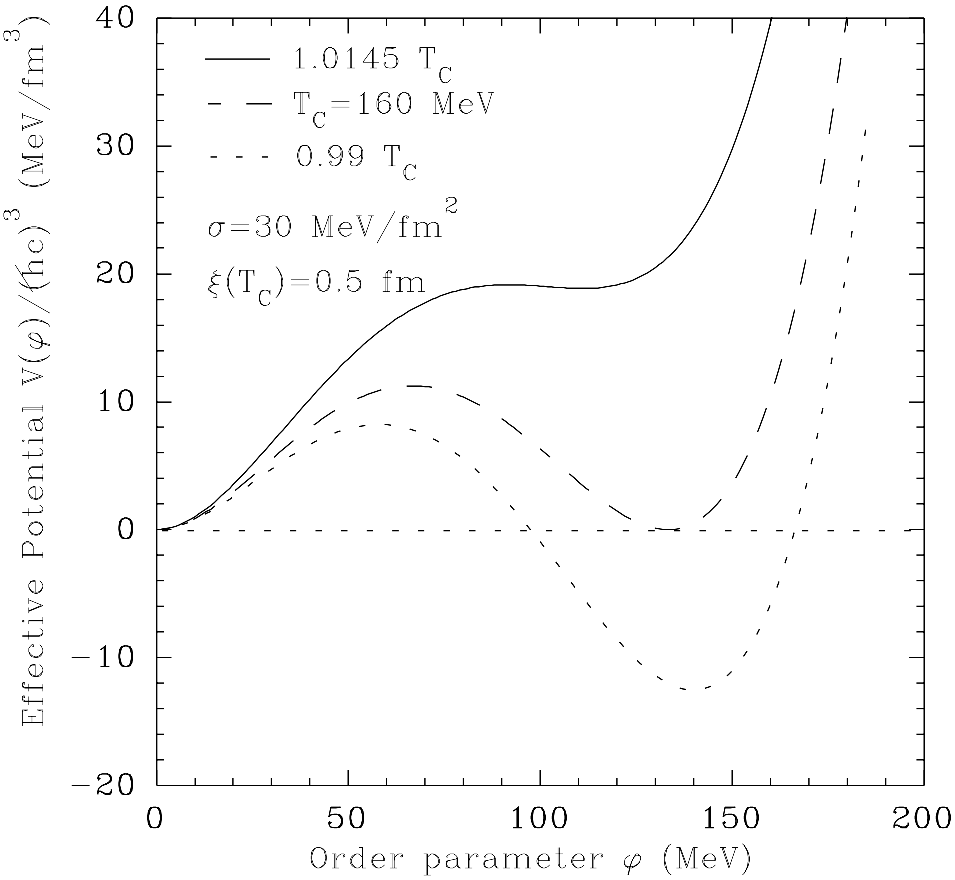
Fig. 3 The nucleation barrier F_C/T for critical bubbles with and without subcritical bubble correction as function of temperature for $\sigma = 50$ MeV/fm^2 is shown in upper panel. Corresponding subcritical hadron fraction γ is shown in the lower panel.

Fig. 4 Same as Fig. 3 but at $\sigma = 30$ MeV/fm^2.

Fig. 5 Same as Fig. 3 but at $\sigma = 10$ MeV/fm^2.

Fig. 6 The temperature variation as function of proper time with and without subcritical bubble correction for σ= 50 MeV/fm^2.

Fig. 7 Same as Fig. 6 but at σ= 30 MeV/fm^2.



1.0145 T_C
T_C=160 MeV
0.99 T_C
σ=30 MeV/fm^2
ξ(T_C)=0.5 fm
Effective Potential V(φ)/(ħc)^3 (MeV/fm^3)
Order parameter φ (MeV)
40
30
20
10
0
−10
−20
0
50
100
150
200

Fig. 1

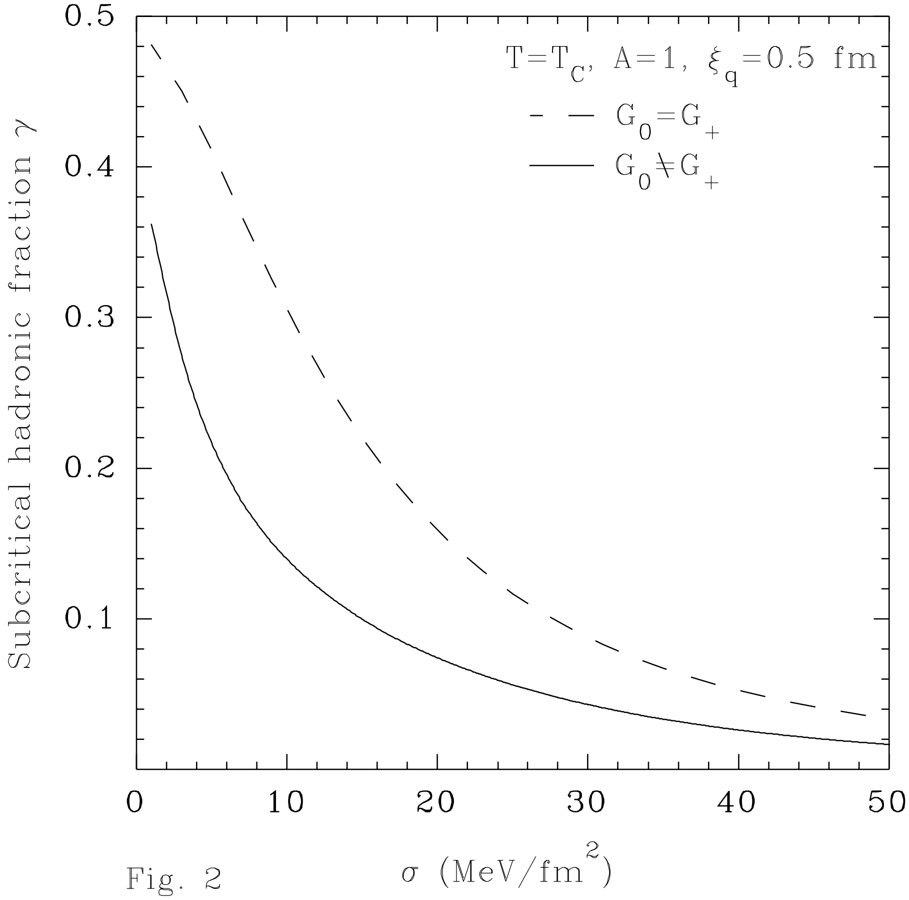
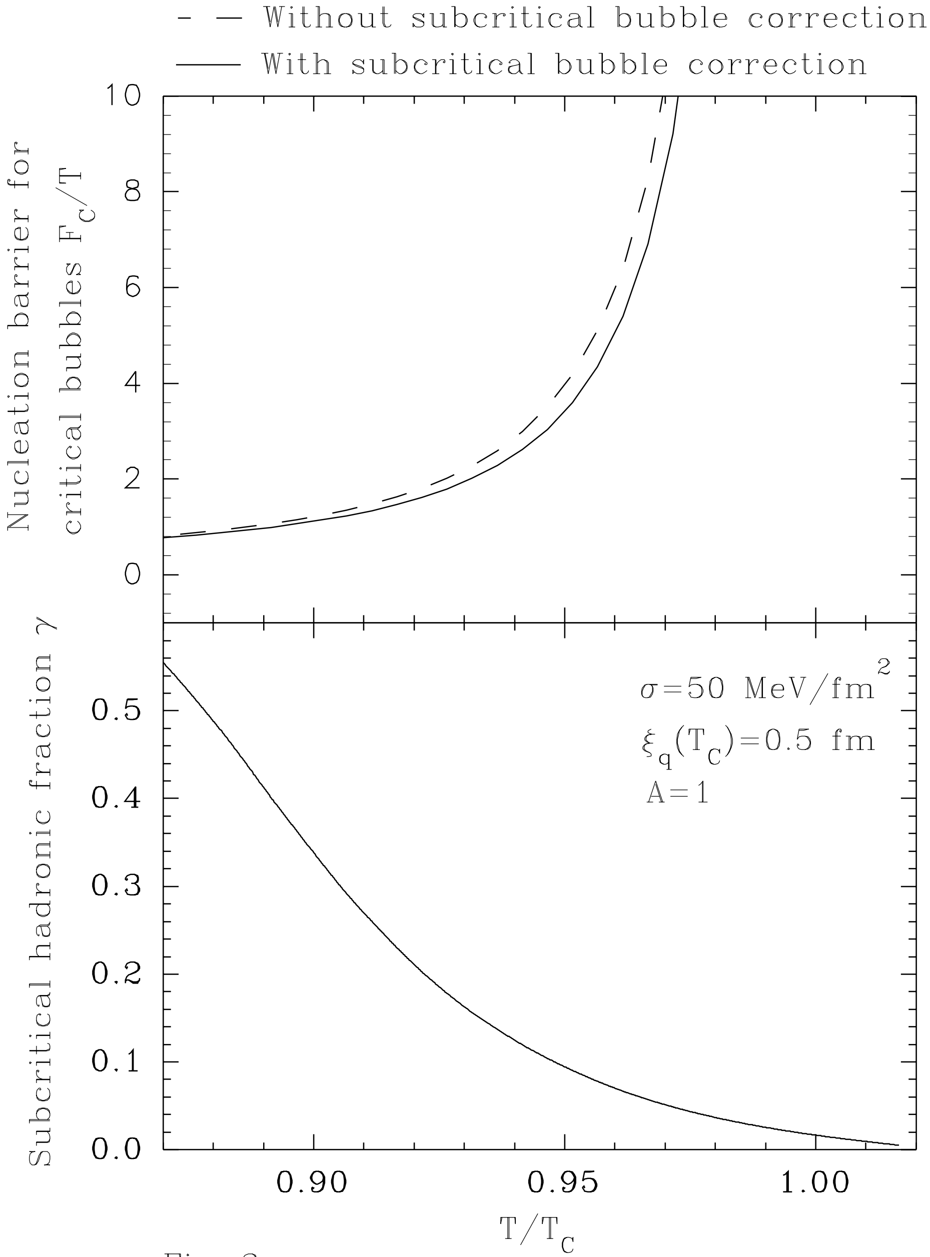
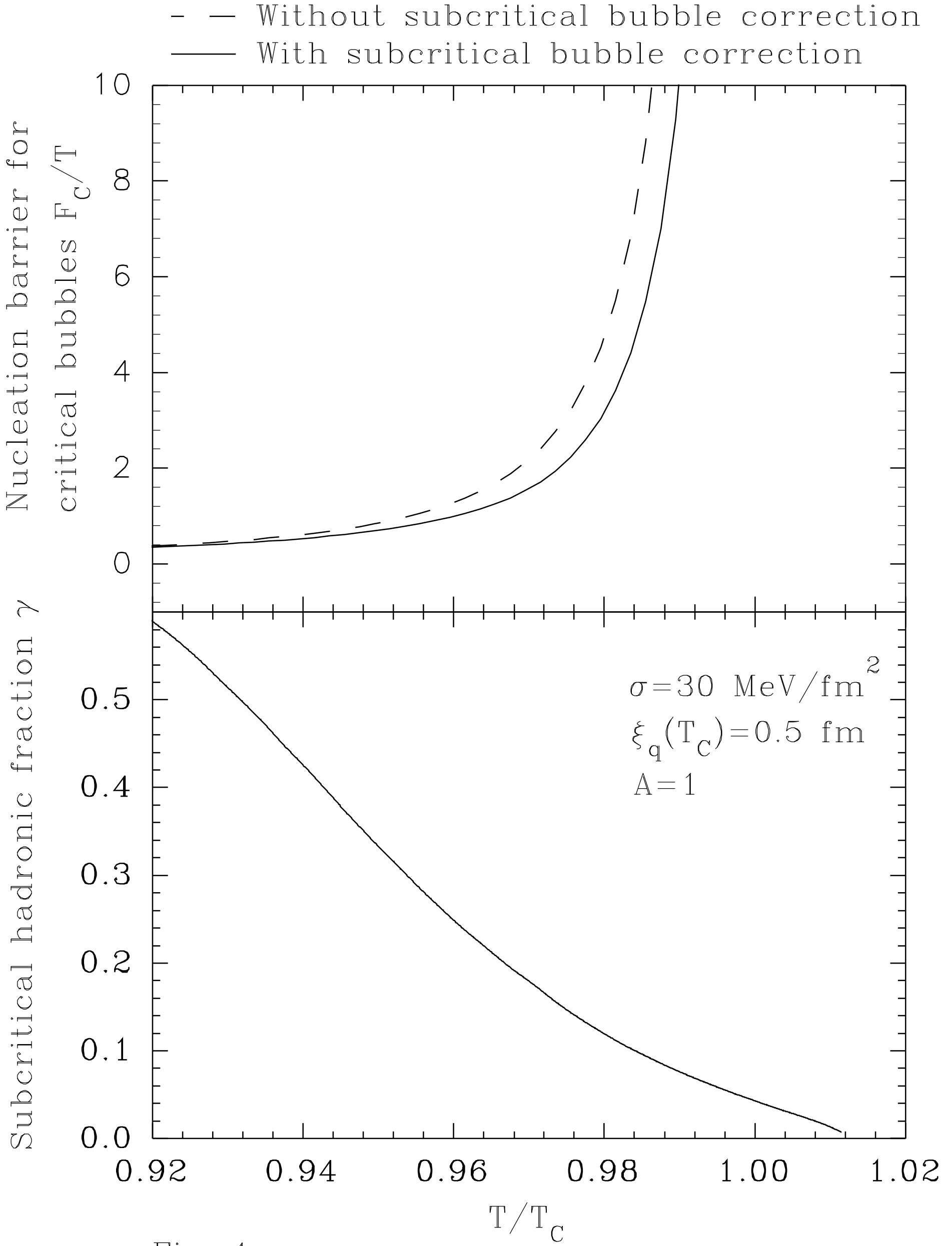

$T=T_C$, $A=1$, $\xi_q=0.5$ fm
$G_0=G_+$
$G_0\neq G_+$
Subcritical hadronic fraction γ
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
σ (MeV/fm^2)

Fig. 2



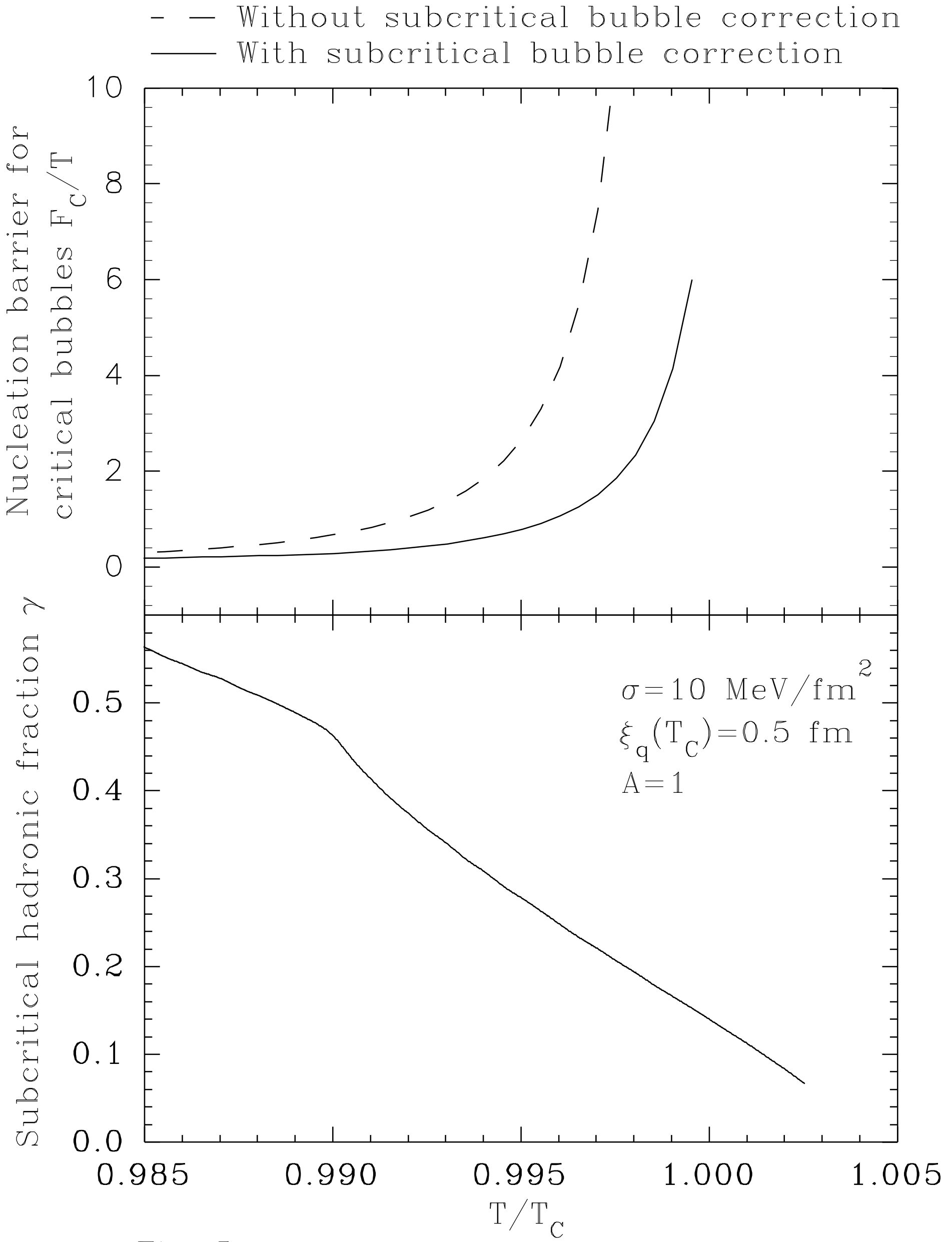
Without subcritical bubble correction
With subcritical bubble correction
Nucleation barrier for critical bubbles F_C/T
Subcritical hadronic fraction γ
σ=50 MeV/fm^2
$\xi_q(T_C)$=0.5 fm
A=1
T/T_C

Fig. 3



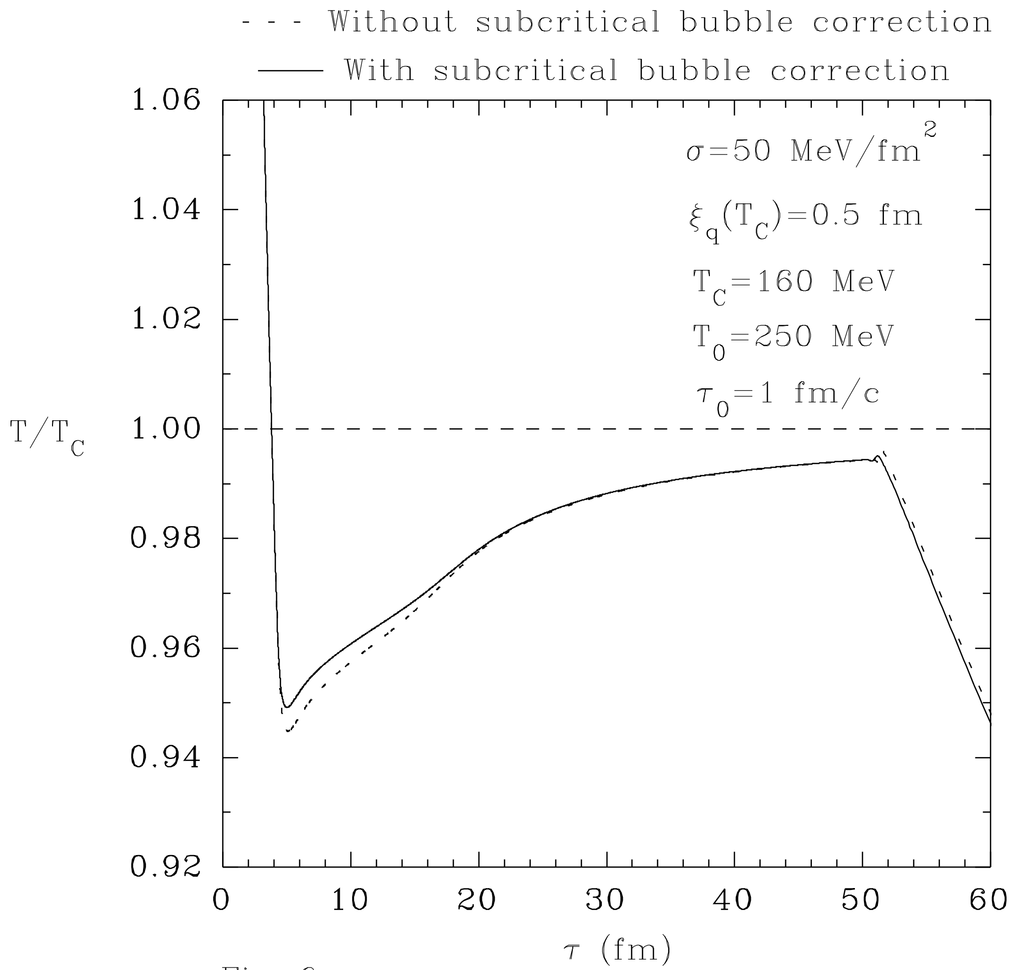
Without subcritical bubble correction
With subcritical bubble correction
Nucleation barrier for critical bubbles F_C/T
Subcritical hadronic fraction γ
σ=30 MeV/fm^2
$\xi_q(T_C)$=0.5 fm
A=1
0.92
0.94
0.96
0.98
1.00
1.02
T/T$_C$

Fig. 4



Without subcritical bubble correction
With subcritical bubble correction
Nucleation barrier for critical bubbles F_C/T
Subcritical hadronic fraction γ
σ=10 MeV/fm^2
$\xi_q(T_C)$=0.5 fm
A=1
T/T_C

Fig. 5



- - - Without subcritical bubble correction
With subcritical bubble correction
σ=50 MeV/fm²
ξq(TC)=0.5 fm
TC=160 MeV
T0=250 MeV
τ0=1 fm/c
T/TC
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0
10
20
30
40
50
60
τ (fm)

Fig. 6

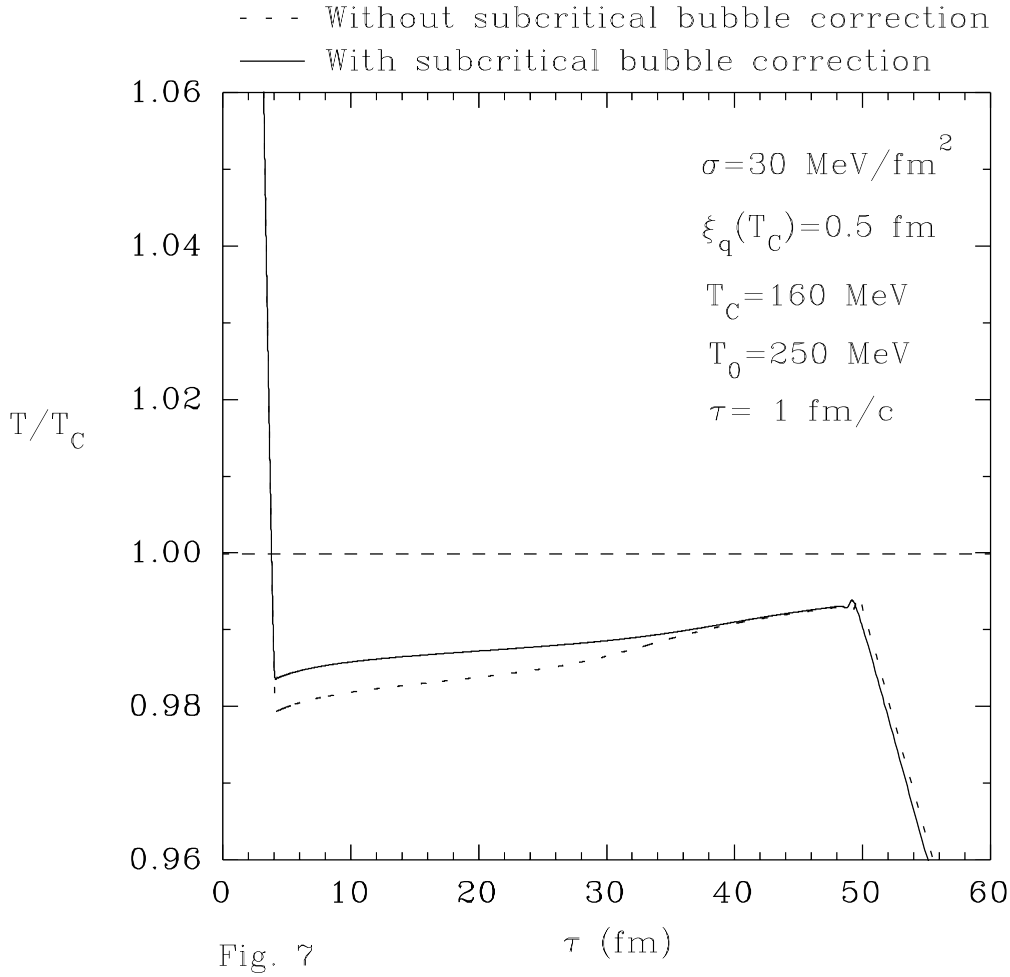
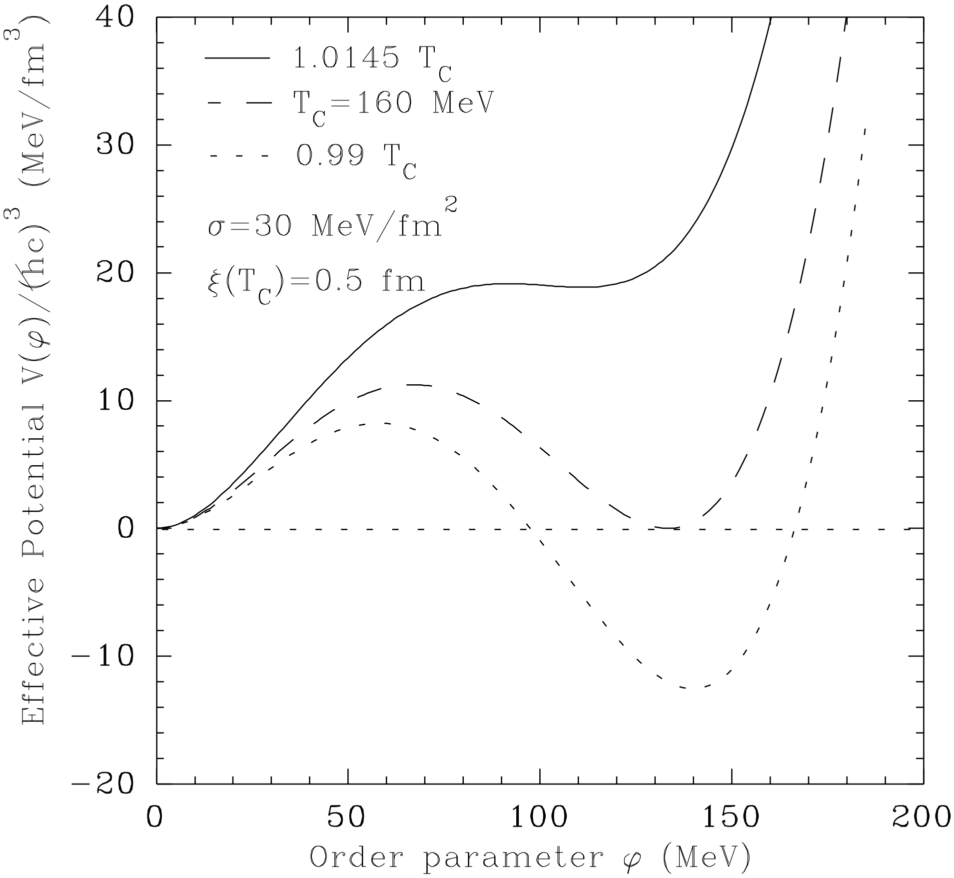
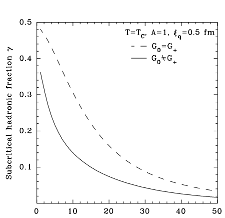

- - - Without subcritical bubble correction
With subcritical bubble correction
σ=30 MeV/fm^2
$\xi_q(T_C)$=0.5 fm
T_C=160 MeV
T_0=250 MeV
τ= 1 fm/c
T/T$_C$
1.06
1.04
1.02
1.00
0.98
0.96
0
10
20
30
40
50
60
τ (fm)

Fig. 7



1.0145 T_C
T_C=160 MeV
0.99 T_C
σ=30 MeV/fm^2
$\xi(T_C)$=0.5 fm
Effective Potential $V(\varphi)/(\hbar c)^3$ (MeV/fm^3)
Order parameter φ (MeV)
40
30
20
10
0
−10
−20
0
50
100
150
200

Fig. 1



Subcritical hadronic fraction γ
$T=T_C$, $A=1$, $\xi_q=0.5$ fm
$G_0=G_+$
$G_0\neq G_+$
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
σ (MeV/fm^2)

Fig. 2

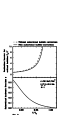

Without subcritical bubble correction
With subcritical bubble correction

Fig. 3